Lattice Determination of the $B^*B\pi$ Coupling *

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The coupling $g_{B^*B\pi}$ is related to the form factor at zero momentum of the axial current between B^* and B states. Moreover it is related to the effective coupling between heavy mesons and pions that appear the heavy meson chiral Lagrangian. This coupling has been evaluated on the lattice using static heavy quarks and light quark propagators determined by a stochastic inversion of the fermionic bilinear. We found the value $g = 0.42(4)(8)$. Beside its theoretical interest, this quantity has phenomenological implications in $B \to \pi + \bar{l}l$ decays.

1. INTRODUCTION

A matrix element with important phenomenological applications is

$$\left\langle B^0(p)\pi^+(q) \mid B^{*+}(p')\right\rangle \tag{1}$$

which can be parametrised in terms of the form factor $g_{BB^*\pi}(q^2)$ in the following way:

$$-g_{B^*B\pi}(q^2)q_\mu\eta^\mu(p')(2\pi)^4\delta^4(p'-p'-q) \tag{2}$$

where $\eta^\mu(p')$ is the polarization vector of the asymptotic state $B^{*+}(p')$.

For on-shell external states, $p' = p+q$, one can perform an LSZ reduction of the pion in eq. (1)

$$i(m_\pi^2 - q^2)\int e^{iqx}\left\langle B(p)\right|\pi(x)\left|B^*(p')\right\rangle \mathrm{d}^3x \tag{3}$$

Using the PCAC definition of the pion in terms of the axial current

$$\pi(x) = \frac{1}{m_\pi^2 f_\pi}\partial^\mu A_\mu(x) \tag{4}$$

one obtains

$$q^\mu\frac{m_\pi^2 - q^2}{m_\pi^2 f_\pi}\int e^{iqx}\left\langle B^0(p)\right| A_\mu(x)\left|B^*(p')\right\rangle \mathrm{d}^3x \tag{5}$$

Since in the limit $q \to 0$, eq. (5) and eq. (2) parametrise the same matrix element of eq. (1), one derives a Goldberger-Treiman like relation for the pion system [1]

$$g_{B^*B\pi}(0) = \frac{2m_B}{f_\pi}g \tag{6}$$

where g is defined as

$$g = \int\frac{\left\langle B(p)\right| A_\mu(x)\left|B^*(p)\right\rangle}{2m_B}\eta^\mu \mathrm{d}^3x \tag{7}$$

The parameter g is also the effective coupling which appears in the the Heavy Meson Chiral Langrangian [2] and can be used to constrain the form factors that appear in the matrix elements of the weak vector current, $\overline{u}\gamma^\mu b$. Which matrix elements are relevant for $B \to \pi + \bar{l}l$ decays. One of the best present techniques to extract V_{ub} is by comparing the experimental values for these form factors with the theoretical ones, obtained by fitting lattice Montecarlo results. Therefore a precise determination of g can be used to reduce the number of free parameters in the fit and, therefore, reduce the theoretical uncertainty on V_{ub}.

Even if the matrix element of eq. (1) does not directly correspond to a physical process, because $B^* \to B+\pi$ is kinematically forbidden, the equivalent process for D systems, $D^* \to D+\pi$, is indeed physical and the effective coupling for this D decay process coincides with our g up to $1/m_c$ corrections.

2. SIMULATION

We have performed our simulations on 20 quenched gauge configurations, generated on a $12^3 \times 24$ lattice at $\beta = 5.7$, corresponding to $a^{-1} = 1.10$ GeV, with a tadpole improved SW action ($c_{SW} = 1.57$). The heavy quark propaga-

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tors are evaluated in the static limit, while the light quark propagators are evaluated performing a stochastic inversion on the fermion matrix Q

$$(Q^{-1})_{ij} = \int [\mathrm{d}\phi](Q_{jk}\phi_k)^* \phi_i e^{-\phi_l^* (Q^\dagger Q)_{lm}\phi_m} \tag{8}$$

and 10 pseudofermionic fields ϕ_i for each gauge configuration were generated by Montecarlo. Moreover the maximal variance reduction technique is implemented to reduce the statistical noise [3]. Two values of κ are considered, $\kappa_1 = 0.13843$ and $\kappa_2 = 0.14077$, corresponding to a light quark mass of 140 MeV and 75 MeV respectively, with a critical value $\kappa_{crit} = 0.14351$. The interpolating operator $J^\dagger$ (J) that create (annihilate) a static B meson is smeared with a two-steps fuzzing procedure for the light quark fields. We indicate the local operators with the superscript L and the fuzzed ones with the superscript F.

We have evaluated the two point correlation function at zero momentum

$$C_2^{FF(FL)}(t) = \overline{\langle 0|\, J(\mathbf{y},0) J^\dagger(\mathbf{y},t)\, |0\rangle} \tag{9}$$

both for two-fuzzed (FF) and one-fuzzed one-local (FL) J operators. The average is on the spatial position $\mathbf{y}$ and it has the effect of increasing the statistics. This has been possible thanks to all-to-all stochastic propagators. From fitting the asymptotic (large t) behaviour of C_2 with a single exponential we have extracted $Z^{L(F)} = \langle 0|\, J^{L(F)}\, |B\rangle / \sqrt{2m_B}$.

Analogously we have computed the three point correlation function at zero momentum

$$C_{3\mu}^{FF}(r,t_1,t_2) = \overline{\langle 0|\, J(\mathbf{y},-t_1) A_\mu(\mathbf{x}+\mathbf{y},0) J^\dagger(\mathbf{y},t_2)\, |0\rangle} \tag{10}$$

for two-fuzzed J operators. Here the average is on both the spatial coordinates $\mathbf{y}$ and $\mathbf{x}$, but keeping fixed $r = |\mathbf{x}|$. For the spatial indices $\mu = 1,2,3$ we also used the rotational invariance of the lattice so that $C_{31} = C_{32} = C_{33}$.

Finally we have computed

$$E^0(r,t) = (Z^F)^2 C_{30}^{FF}(r,t,t) / \left[C_2^{FF}(t)\right]^2 \tag{11}$$

and

$$\overline{E}(r,t) = (Z^F)^2 C_{31}^{FF}(r,t,t) / \left[C_2^{FF}(t)\right]^2 \tag{12}$$

These two functions of r are plotted in fig. 1 for the lightest value of κ and values of $t = 3,4,5,6$. As expected $E^0(r,t) \simeq 0$, because the polarization of a static B^* meson is ortogonal to the temporal direction, while the averaged spatial component $\overline{E}(r,t)$ is almost independent on t and presents an exponential decay in r (this is confirmed by a number of different fitting tests and by visualizing the points in log scale). In fact in the asymptotic regime in t

$$\overline{E}(r,t) = \langle B|\, A_\mu(r)\, |B^*\rangle / (2m_B) \tag{13}$$

Going back to the definition, eq. (7), the lattice regularised g coupling can be extracted from the spatial integral of $f(r) = Se^{-r/r_0}$, the function fitting $\overline{E}(r,t)$. We find

$$g^{\mathrm{latt}} = \int f(r)(4\pi r^2)\mathrm{d}r = \begin{cases} 0.54(5) & \text{for } \kappa_1 \\ 0.53(5) & \text{for } \kappa_2 \end{cases} \tag{14}$$

We observe that the main source of error in g^{latt} is the lattice breaking of rotational invariance, in fact the distribution of the points $\overline{E}(r)$ around the fitting function $f(r)$ exhibits a regular pattern which is independent on t and κ.

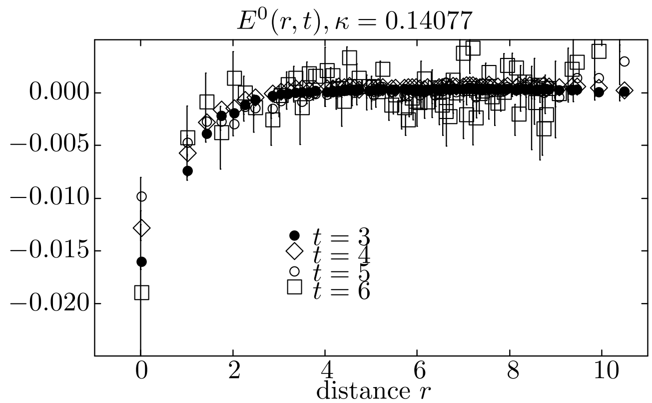
3. MATCHING

From C_2 we are able to extract the B meson decay constant (in the static approximation), using the relation

$$f_B^{\mathrm{static}} = Z_A^{\mathrm{static}} \sqrt{\frac{2}{m_B}} Z^L a^{-3/2} \tag{15}$$

For our lattice $Z_A^{\mathrm{static}} = 0.78$ is computed using the Lepage-Mackenzie procedure [4] of matching lattice vs continuum at a best scale, which in our case is determined to be $q^* a = 2.29$. Our result is $f_B^{\mathrm{static}} = 0.43(1)(8)$ GeV. We observe that a one-mass fit gives a value of f_B^{static} 20% bigger than the value extracted from a three-masses fit [3]. In fact we obtain a value for this quantity that lies above other lattice calculations of the same quantity. We take into account this effect by adding a systematic error to our results (in particular to g) of the order 20%.

The quantity we are interested in is the g coupling renormalized in the $\overline{\mathrm{MS}}$ scheme at the m_B



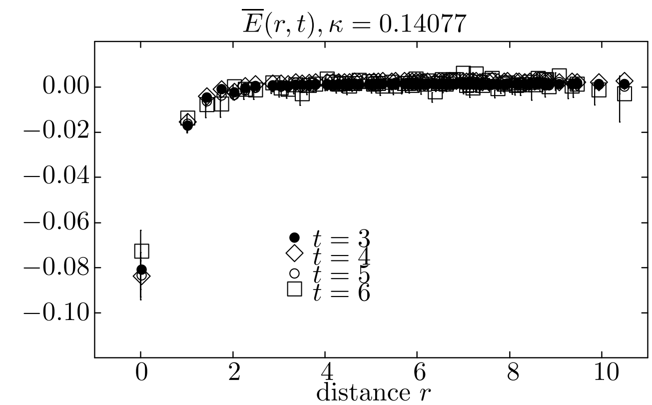


Figure 1. Plots of $E^0(r,t)$ and $\bar{E}(r,t)$ as funtions of r for different values of t $(= 3, 4, 5, 6)$ for κ_2.

scale

$$g = Z_A^{\text{tadpole}} g^{\text{latt}} = Z_A^{\text{1-loop}} \frac{u_0}{u_0^{\text{1-loop}}} g^{\text{latt}} \tag{16}$$

where $Z_A^{\text{1-loop}} = 1 - 13.8\frac{\alpha C_F}{4\pi}$, $u_0^{\text{1-loop}} = 1 - \pi^2\frac{\alpha C_F}{4\pi}$ and $u_0 = 0.86081$ is the average plaquette. These values imply $Z_A^{\text{tadpole}} = 0.806$.

4. CONCLUSIONS

Substituting the values of eq. (14) into eq. (16) we obtain

$$g = \begin{cases} 0.44(4) & \text{for } \kappa_1 \\ 0.43(4) & \text{for } \kappa_2 \end{cases} \tag{17}$$

Performing a naive extrapolation to the chiral limit and including our evaluation for the systematic error we summarise our result as

$$g = 0.42(4)(8) \tag{18}$$

This number has to be compared to the best estimate from a global analysis of available results [2]

$$g \simeq 0.38 \tag{19}$$

and with an estimate obtained from lattice data for the semileptonic B decay form factor (assuming Vector Meson Dominance),

$$g = 0.50(5)(10) \tag{20}$$

A recent, independent, lattice analysis confirmed our result [5].

One remark is in order. We consider this study an exploratory one because of the small lattice, the large lattice spacing, the poor chiral extrapolation and the use of a non-standard pioneering technique such as stochastic propagators. As we observed, the g parameter has important phenomenological applications and we believe our study shows that this number can be evaluted on the lattice. A better determination is required and is feasible with present computing power.

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